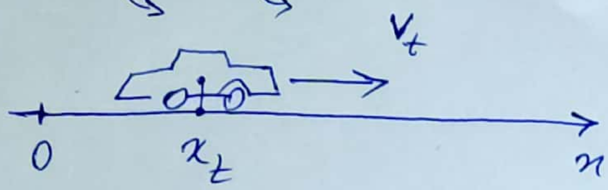
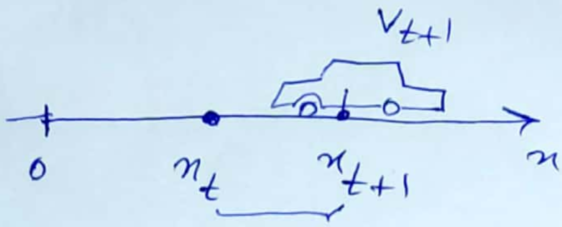
$P(C \mid \text{Parents})$





سرعت ثابت

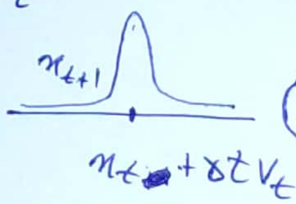
$$v_{t+1} = v_t$$

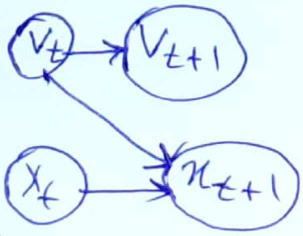
$$x_{t+1} = x_t + \Delta t\, v_t \implies \begin{aligned} v_{t+1} &\simeq v_t \\ x_{t+1} &\simeq x_t + \Delta t\, v_t \end{aligned}$$

$\Downarrow$

$$v_{t+1} \simeq v_t \implies$$

$$x_{t+1} \simeq x_t + \Delta t\, v_t$$





$P(x_{t+1} \mid x_t, v_t)$

---

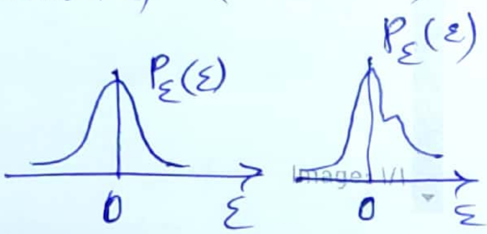
noise Model

$\cancel{v_{t+1} = v_t + \varepsilon_v}$

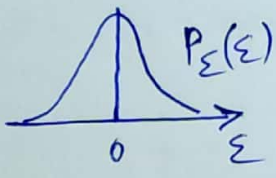
$X \to Y \quad P(Y \mid X)$

$$Y = \underbrace{f(X)}_{\text{(deterministic)}} + \varepsilon \rightarrow \text{noise (stochastic)}$$

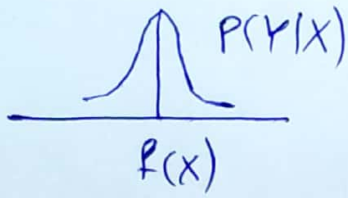
noise: Random variable $\varepsilon \propto P_\varepsilon$

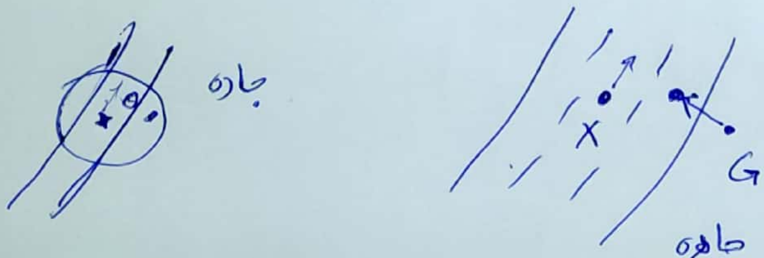


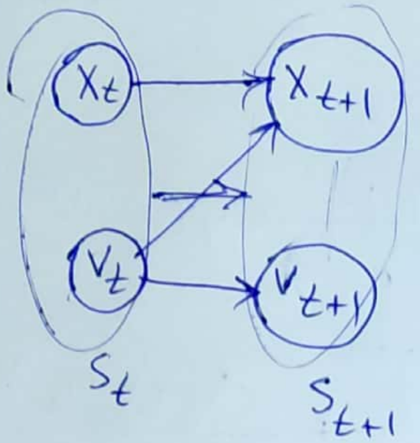
$$Y = f(X) + \varepsilon \implies \varepsilon = Y - f(X)$$



$$P(Y \mid X) = P_\varepsilon(Y - f(X))$$







$$S_t = \begin{bmatrix} X_t \\ V_t \end{bmatrix}$$

We are interested in

$P(S_{t+1} \mid S_t) \to$ transition model

$$= P(X_{t+1}, V_{t+1} \mid X_t, V_t)$$

$$= Pr(X_{t+1} = X_t + \Delta t V_t + \varepsilon_v, V_{t+1} = V_t + \varepsilon_x)$$

$$\boxed{\begin{aligned} V_{t+1} &= V_t + \varepsilon_v \\ X_{t+1} &= X_t + \Delta t \, V_t + \varepsilon_x \end{aligned}}$$

$$= \cancel{Pr(\varepsilon_x = X_{t+1} + \Delta t V_t, \; \varepsilon_v = X_{t+1} - V_t)}$$

$\cancel{P(X_{t+1}} \mid$

$$P(S_{t+1} \mid S_t) = P(X_{t+1}, V_{t+1} \mid X_t, V_t)$$

$$= Pr(\varepsilon_x = X_{t+1} - X_t - \Delta t V_t, \quad \varepsilon_v = V_{t+1} - V_t)$$

prob. density

$P_\varepsilon(\varepsilon_x, \varepsilon_v)$ joint distribution of $\varepsilon_x, \varepsilon_v$

if $\varepsilon_x, \varepsilon_v$ are independent $P_\varepsilon(\varepsilon_x, \varepsilon_v) = P_{\varepsilon_x}(\varepsilon_x) P_{\varepsilon_v}(\varepsilon_v)$

$$\Longrightarrow P(S_{t+1} \mid S_t) = P_\varepsilon(\varepsilon_x = X_{t+1} - X_t - \Delta t V_t) P_v(V_{t+1} - V_t)$$

$$= P(X_{t+1}, V_{t+1} \mid X_t, V_t) = P(X_{t+1} \mid X_t, V_t) \, P(V_{t+1} \mid V_t)$$

$$= \frac{1}{\sigma_x \sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{X_{t+1} - X_t - \Delta t V_t}{\sigma_\varepsilon}\right)^2\right) \frac{1}{\sigma_v \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{V_{t+1} - V_t}{\sigma_v}\right)^2}$$

$$P(X_{t+1}, V_{t+1} \mid X_t, V_t)$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma_X} \exp\Big(\frac{-1}{2\sigma_X^2}\big(X_{t+1} - \overbrace{(X_t + \Delta_t V_t)}^{\mu_X}\big)^2\Big) \frac{1}{\sqrt{2\pi}\,\sigma_V} \exp\Big(\frac{-1}{2\sigma_V^2}\big(V_{t+1} - \overbrace{V_t}^{\mu_V}\big)^2\Big)$$

$$= \frac{1}{(\sqrt{2\pi})^2\,\sigma_X \sigma_V} \exp\Big(-\frac{1}{2}\Big[\frac{(X_{t+1}-\mu_X)^2}{\sigma_X^2} + \frac{(V_{t+1}-\mu_V)^2}{\sigma_V^2}\Big]\Big)$$

$$= \frac{1}{(\sqrt{2\pi})^2\,\sigma_X \sigma_V} \exp\Big(-\frac{1}{2}\underbrace{[X_{t+1}-\mu_X,\ V_{t+1}-\mu_V]}_{2\times 1} \underbrace{\begin{bmatrix} \sigma_X^{-2} & 0 \\ 0 & \sigma_Y^{-2} \end{bmatrix}}_{2\times 2} \underbrace{\begin{bmatrix} X_{t+1}-\mu_X \\ V_{t+1}-\mu_V \end{bmatrix}}_{2\times 1}\Big)$$

$$\Sigma = \begin{bmatrix} \sigma_X^2 & 0 \\ 0 & \sigma_Y^2 \end{bmatrix} \quad \mu = \begin{bmatrix} \mu_X \\ \mu_Y \end{bmatrix} \quad S_{t+1} = \begin{bmatrix} X_{t+1} \\ V_{t+1} \end{bmatrix}$$

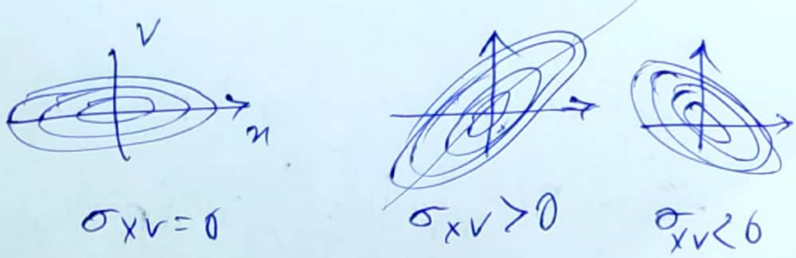
$$= \frac{1}{(\sqrt{2\pi})^d \det(\Sigma)^{1/2}} \exp\Big(-\frac{1}{2}(S_{t+1}-\mu)^T \Sigma^{-1} (S_{t+1}-\mu)\Big)$$

$d = 2$

تعداد متغیرها

→ general case for when $\varepsilon_X, \varepsilon_V$ are ~~not~~ dependent

$$\Sigma = \begin{bmatrix} \sigma_X^2 & \sigma_{XV} \\ \sigma_{XV} & \sigma_V^2 \end{bmatrix}$$



$\sigma_{XV} = 0$ $\quad$ $\sigma_{XV} > 0$ $\quad$ $\sigma_{XV} < 0$